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Research Article

Nano $g^*\alpha$ -continuous functions in nano topological spaces

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Abstract

The aim of this paper is to introduce and study the concept of new class of function called nano generalized star alpha -continuous functions in nano topological spaces. Some of the basic properties of nano generalized star alpha -continuous functions are analyzed.

Keywords: $Ng^*\alpha$ -closed sets, $Ng^*\alpha$ -continuous functions, $Ng^*\alpha$ -irresolute functions.

1. Introduction

Continuity of functions is one of the core concepts of topology. In general, a continuous function is one, for which small changes in the input result is in small changes in the output. The concept of nano topology was introduced by M. Lellis Thivagar and Carmel Richard [4] introduced which is defined in terms of lower and upper approximations of X. The elements of nano topological space are called nano open sets. He has defined nano closed sets, nano interior and nano closure of a set in nano topological spaces. He has also introduced a nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphisms in nano topological spaces.

The structure of this manuscript is as follows:

In section 2, we recall some existing definitions which are more important to prove our main results.

In section 3, we introduce and study some theorems which satisfies the condition of Ng^* -continuous functions.

In section 4, we introduce and examine some theorems which satisfies the conditions of $Ng^*\alpha$ - irresolute functions.

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2. Preliminaries

In this section, we recall some basic definitions and results in nano topological spaces are given, which are useful to prove the main results.

Definition 2.1. [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
That is, $L_R(X) = \{\bigcup_{x\in U} \{ R(x) : R(x) \subseteq X\}\}$, where R(x) denotes the equivalence class determined by x ∈ U.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$.
That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classifed as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. [4] If (U, R) is an approximation space and X, Y ⊆ U, then

(1) $L_R(X) \subseteq X \subseteq U_R(X)$

(2) $L_R(\phi) = U_R(\phi) = \phi$

(3) $L_R(U) = U_R(U) = U$

(4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y

(9) $U_R(X^c) = [(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$

(11) $L_R[L_R(X)] = U_R[LR(X)] = L_R(X)$

Definition 2.3. [4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. We call

{U, $\tau_R(X)$} is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets.

Remark 2.4. [4] If $[\tau_R(X)]$ is the nano topology on U with respect to X. Then the set B = {U, $\tau_R(X)$, $B_R(X)$} is the basis for $[\tau_R(X)]$.

Definition 2.5. [4] If (U, $\tau_R(X)$) is a nano topological space with respect to X. Where X ⊆ U and if A ⊆ U. Then

(i) The nano interior of the set A is defined as the Union of all nano open subsets contained in A and is denoted by Nint(A). Nint(A) is the largest nano open subset of A.

(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

Definition 2.6. A function f : (U,$\tau_R(X)$) → (V, $\tau_{R'}(X)$) is called

- Nano -continuous [3], if $f^{-1}(V)$ is nano -open in (X, τ) for every nano open set V in (Y, σ).
- Nano pre -continuous [8], if f^{-1} (V) is nano pre -open in (X, τ) for every nano open set V in (Y, σ).
- Nano α -continuous [8], if $f^{-1}(V)$ is nano α -open in (X, τ) for every nano open set V in (Y, σ).
- Nano regular -continuous [10], if $f^{-1}(V)$ is nano regular -open in (X, τ) for every nano open set V in (Y, σ).

Definition 2.7. If (U, $\tau_R(X)$) is a nano topological space and if A ⊆ U. Then A is said to be

(1) Ngp -closed [2], if Npcl(A) ⊆ G whenever A ⊆ G and G is nano -open in U.

(2) Ngpr -closed [2], if Npcl(A) ⊆ G whenever A ⊆ G and G is nano regular -open in U.

(3) Nsg -closed [1], if Nscl(A) ⊆ G whenever A ⊆ G and G is nano semi -open in U.

(4) Ngs -closed [1], if Nscl(A) ⊆ G whenever A ⊆ G and G is nano -open in U.

(5) Ngsp -closed [1], if Nspcl(A) ⊆ G whenever A ⊆ G and G is nano -open in U.

Definition 2.8. [5] A function f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nsg -continuous if $f^{-1}(V)$ is Nsg -closed in (X, τ) for every nano -closed set V in (Y, σ).

Definition 2.9. [8] A function f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngs -continuous if $f^{-1}(V)$ is Ngs -closed in (X, τ) for every nano -closed set V in (Y, σ).

Definition 2.10. [8] A function f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngpr -continuous if $f^{-1}(V)$ is Ngpr -closed in (X, τ) for every nano -closed set V in (Y, σ).

3. Nano $g^*\alpha$ -continuous functions

In this section we define and study the new class of functions, namely Ng*α -continuous functions in nano topological spaces.

Definition 3.1. Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be a nano topological spaces. Then the function f : (U, $\tau_R(X)$)→ (V, $\tau_{R'}(Y)$) is said to be nano generalized star α -continuous (briefly Ng*α -continuous) on U if the inverse image of every nano -open set in V is Ng*α -open set in U.

Example 3.2. Let U = {a, b, c, d}, with U/R = {{a},{b, c}, {d}} and X = {b, d}. Then
$\tau_R(X)$ = {U, ϕ, {d}, {b, c}, {b, c, d}} and $\tau_R(X)^c$= {U, ϕ, {a, b, c}, {a, d}, {a}}. Also V = {x, y, z, w} with V/R$'$ = {{y}, {z},{x, w}} and Y = {y, w}.
Then $\tau_{R'}(Y)$ = {U, ϕ, {y}, {x, w}, {x, y, w}}, and $\tau_{R'}(Y)^c$= {V, ϕ, {x, z, w}, {y, z}, {z}}. Define f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) as f (a) = z, f (b) = x, f (c) = w and f (d) = y. Then f^{-1}(y) = d, f^{-1}({x, w}) = {b, c}, f^{-1}({x, y, w}) = {b, c, d} and f^{-1}(V) = U. That is the inverse image of every nano -open set in V is Ng*α -open set in U. Therefore f is Ng*α -continuous.

Theorem 3.3. A function f : (U, $\tau_R(X)$)→ (V, $\tau_{R'}(Y)$) is said to be Ng*α -continuous if the inverse image of every nano -closed set in V is Ng*α -closed set in U.

Proof. Let f be Ng*α -continuous and F be nano closed set in V . That is V −F is nano open set in V. Since f is Ng*α- continuous, f^{-1}(V −F) is Ng*α -open set in U . That is f^{-1} (V)− f^{-1} (F) = U −f^{-1} (F) is Ng*α -open set in U . Hence f^{-1} (F) is Ng*α -closed set in U, if f is Ng*α -continuous on U.

Conversely, let us assume that the inverse image of every nano -closed set in V is Ng*α -closed set in U . Let G be nano open set in U. Then V − G is nano closed set in V. That is f^{-1} (V) − f^{-1} (G) = U − f^{-1} (G) is Ng*α -closed set in U . Hence f^{-1} (G) is Ng*α -open set in U. That is the inverse image of every nano -open set in V is Ng*α -open set in U. That is f is Ng*α -continuous on U .

Theorem 3.4. A function f : (U, $\tau_R(X)$)→ (V, $\tau_{R'}(Y)$) is Ng*α -continuous iff f (Ncl(A)) ⊆ Ncl(f(A)) for every subset A of U.

Proof. Let f be $Ng^*\alpha$ -continuous and $A \subseteq U$. Then f(A) $\subseteq V$. Also $Ncl(f(A))$ is nano closed in V. Since f is $Ng^*\alpha$ -continuous, f^{-1}(Ncl(f(A))) is Ng*α -closed set containing A. But every nano closed set is Ng*α -closed set in U and is the smallest nano closed set containing A. Therefore Ncl(A) ⊆ f^{-1} (Ncl(f (A))). (ie) f (Ncl(A)) ⊆ Ncl(f(A)).

Conversely, let f((Ncl(A)) ⊆ Ncl(f (A)) for every closed subset A of U . If F is nano closed set in V and Since f^{-1}⊆ U . We have f (Ncl(f^{-1} (F))) ⊆ Ncl(f (f^{-1} (F))) = Ncl(F). That is Ncl(f^{-1} (F)) ⊆ (f^{-1}Ncl(F)) = f^{-1} (F), Since F is nano closed set in V. Thus Ncl(f^{-1} (F)) ⊆ f^{-1} (F). But every nano closed set is Ng*α -closed set in

U, we have $f^{-1}(F)$ is $Ng^*\alpha$ -closed set in V. Hence f is $Ng^*\alpha$ -continuous on U.

Remark 3.5. If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -continuous, Then f (Ncl(A)) is not necessarily equal to Ncl(f (A)), where $A \subseteq U$.

Example 3.6. Let U = {a, b, c, d}, with U/R={{a},{b, c}, {d}}, and X = {b, d}. Then $\tau_R(X)$ ={U, ϕ, {d},{b, c}, {b, c, d}}. Let V = {x, y, z, w} with V/R' = {{y}, {z}, {x, w}} and Y = {y, w}. Then $\tau_{R'}(Y)$ ={V, ϕ, {y}, {x, w}, {x, y, w}}. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ by f(a) = z, f (b) = x, f (c) = w and f (d) = y. Then $\tau_R(X)^c$ = {U, ϕ, {a, b, c}, {a, d}, {a}} and $\tau_R(Y)^c$ = {V, ϕ, {x, z, w},{y, z}, {z}} are the complements of $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$. Now f^{-1} (x, z, w) = {a, c, d}, f^{-1} ({y, z}) = {a, b}, and f^{-1} ({z}) = {a}. Therefore the inverse image of every nano closed set in V is $Ng^*\alpha$ -closed set on U. Hence $Ng^*\alpha$ -continuous on U. Let A = {b} $\subseteq$ U. Then f (Ncl(A)) = f ({a, b, c}) = {y, z, w}. But Ncl(f (A)) = Ncl(f {y}) = V . Thus f (Ncl(A)) $\neq$ Ncl(f (A)), even though f is $Ng^*\alpha$-continuous. That is f (Ncl(A)) is not necessarily equal to Ncl(f (A)) where $A \subseteq U$ if f is $Ng^*\alpha$ -continuous on U.

Theorem 3.7. Every nano continuous function is $Ng^*\alpha$ -continuous function.

Proof. Let $f : (U, \tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ be a nano continuous function and A be nano closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is nano closed in $(U, \tau_R(X))$. Since every nano closed is $Ng^*\alpha$ -closed set, $f^{-1}(A)$ is $Ng^*\alpha$ -closed set in $(U, \tau_R(X))$. Hence f is $Ng^*\alpha$ -continuous function.
The converse of the above theorem need not be true as seen from the fallowing example.
Q

Example 3.8. Let U = {a, b, c, d}, X = {b, d} and U/R = {{a},{b, c}, {d}}. Then $\tau_R(X)$ = {U, ϕ, {d},{b, c}, {b, c, d}} and $\tau_R(X)^c$ = {U, ϕ, {a, b, c}, {a, d}, {a}}. Also V = {x, y, z, w}, Y = {y, w} and V/R' ={{y},{z},{x, w}}. Then $\tau_{R'}(Y)$ = {U, ϕ, {y},{x, w},{x, y, w}}, and $\tau_{R'}(Y)^c$ = {V, ϕ, {x, z, w},{y, z}, {z}}. Define $f : (U, \tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ as f (a) = y, f (b) = x, f (c) = w and f(d) = z, then f is $Ng^*\alpha$ -continuous function. But not nano continuous function, since f^{-1} ({x, y, w}) = {a, c, b} is not nano closed in $(U, \tau_R(X))$.

Theorem 3.9. Every $N\alpha$ -continuous function is $Ng^*\alpha$ -continuous function.

Proof. Let $f : (U, \tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ be a $N\alpha$ -continuous function and A be $N\alpha$ -closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $N\alpha$- closed in $(U, \tau_R(X))$. Since every $N\alpha$- closed is $Ng^*\alpha$ -closed set, $f^{-1}(A)$ is $Ng^*\alpha$ -closed set in $(U, \tau_R(X))$.
Hence f is $Ng^*\alpha$ -continuous function.
The converse of the above theorem need not be true as seen from the fallowing example. Q

Example 3.10. From the example 3.8, f is $Ng^*\alpha$ -continuous function but not $N\alpha$ -continuous function.

Theorem 3.11. Every $Ng^*\alpha$- continuous function is Ngpr -continuous function.

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Ng^*\alpha$ -continuous function and A be $Ng^*\alpha$ -closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Ng^*\alpha$ -closed in $(U,\tau_R(X))$. Since every $Ng^*\alpha$ -closed is Ngpr -closed set, $f^{-1}(A)$ is Ngpr -closed set in $(U,\tau_R(X))$. Hence f is $Ng^*\alpha$ -continuous function.
Q

Example 3.12. From the example 3.8, Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=w$, $f(b)=y$, $f(c)=x$ and $f(d)=z$, then f is Ngpr -continuous function, since $f^{-1}(\{y,z\})=\{b,d\}$ and
$f^{-1}(\{z\})=\{b\}$ is not $Ng^*\alpha$ -continuous function .

Theorem 3.13. Every $Ng^*\alpha$ -continuous function is Ngs -continuous function.

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Ng^*\alpha$ -continuous function and A be $Ng^*\alpha$ -closed set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Ng^*\alpha$ - closed in $(U,\tau_R(X))$.
Since every $Ng^*\alpha$ -closed is Ngs -closed set, $f^{-1}(A)$ is Ngs -closed set in $(U,\tau_R(X))$. Hence f is $Ng^*\alpha$ -continuous function.

Example 3.14. From the example 3.8, Let us define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=w$, $f(b)=y$, $f(c)=x$ and $f(d)=z$, then f Ngs is continuous function. Since $f^{-1}(\{z\})=\{d\}$ and $f^{-1}(\{y,z\})=\{b,d\}$ is not $Ng^*\alpha$ -continuous function.

Theorem 3.15. Every $Ng^*\alpha$ -continuous function is Nsg -continuous function.

Proof. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $Ng^*\alpha$ -continuous function and A be $Ng^*\alpha$ -closed
set in $(V,\tau_{R'}(Y))$. Then the inverse image of A under the map f is $Ng^*\alpha$ -closed in $(U,\tau_R(X))$.
Since every $Ng^*\alpha$ -closed is Nsg -closed set $f^{-1}(A)$ is Nsg -closed set in $(U,\tau_R(X))$. Hence f is $Ng^*\alpha$ -continuous function.
Q

Example 3.16. From the example 3.8, Let us define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=w$, $f(b)=y$, $f(c)=x$ and $f(d)=z$, then f Nsg is continuous function. Since $f^{-1}(\{z\})=\{d\}$ and $f^{-1}(\{y,z\})=\{b,d\}$ is not $Ng^*\alpha$ -continuous function.

Remark 3.17. Composition of two $Ng^*\alpha$ -continuous function need not be a $Ng^*\alpha$ -continuous function.

Example 3.18. Let $U=V=W=\{a,b,c,d\}$, with $\tau_R(X)=\{U,\phi,\{d\},\{b,c\},\{b,c,d\}\}$, $\tau_{R'}(Y)=\{V,\phi,\{b\},\{a,d\},\{a,b,d\}\}$, $\tau_{R''}(Y)\ (Z)=\{W,\phi,\{a\},\{c,d\},\{a,c,d\}\}$, Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=c$, $f(b)=b$, $f(c)=a$ and $f(d)=d$, and $g:(V,\tau_R(Y))\to(W,\tau_{R''}(Z))$ be the identity map. Then f and g are nano $g^*\alpha$ -

continuous function. But their composition $g\circ f:(U,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is not nano $g^*\alpha$ -continuous because $F = \{a\}$ is nano closed in $(W,\tau_{R''}(Z))$ but $(g\circ f)^{-1}(F) = f^{-1}[g^{-1}(F)] = f^{-1}[g^{-1}(a)] = [f^{-1}\{a\}] = \{c\}$ Which is not nano $g^*\alpha$ -closed in $(U, \tau_R(X))$. Hence the composition of nano $g^*\alpha$ -continuous maps need not be nano $g^*\alpha$ -continuous.

Theorem 3.19. A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -continuous function if and only if $Ng^*\alpha cl(f^{-1}(B))\subseteq f^{-1}(Ncl(B)$ for every subset B of $(V,\tau_{R'}(Y))$.

Proof. Let $B\subseteq V$ and $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -continuous. Then $Ncl(B)$ is nano closed in $(V,\tau_{R'}(Y))$ and hence $f^{-1}(Ncl(B)$ is $Ng^*\alpha$ -closed in $(U, \tau_R(X))$. Therefore, $Ng^*\alpha cl(f^{-1}(Ncl(B))) = f^{-1}(Ncl(B))$. Since $B\subseteq Ncl(B)$, then $f^{-1}(B)\subseteq f^{-1}(Ncl(B)$. That is $Ng^*\alpha cl(f^{-1}(B)\subseteq Ng^*\alpha cl, (f^{-1}(Ncl(B)))= f^{-1}(Ncl(B))$. Hence $Ng^*\alpha cl(B))\subseteq f^{-1}(Ncl(B))$.

Conversely, Let $Ng^*\alpha cl(f^{-1}(B))\subseteq f^{-1}(Ncl(B))$. for every subset $B\subseteq V$. Now let B be nano closed in $(V,\tau_{R'}(Y))$, then $Ncl(B) = B$. Given $Ng^*\alpha cl(f^{-1}(Ncl(B)) = f^{-1}(Ncl(B))$. Hence $Ng^*\alpha cl(f^{-1}(B)) = f^{-1}(B)$. Thus $f^{-1}(B)$ is $Ng^*\alpha$ -closed set in $(U, \tau_R(X))$ for every nano closed set B in $(V,\tau_{R'}(Y))$. Hence $f:(U, \tau_R(X))\to(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -continuous. Q

Theorem 3.20. A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y)$ is $Ng^*\alpha$ -continuous function if and only if $f^{-1}(Nint(B))\subseteq Ng^*\alpha\ int(f^{-1}(B))$ for every subset B of $(V,\tau_{R'}(Y))$.

Proof. Let $B\subseteq V$ and $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be $Ng^*\alpha$- continuous. Then $Ncl(B)$ is nano closed in $(V,\tau_{R'}(Y))$ and hence $f^{-1}(Ncl(B)$ is $Ng^*\alpha$ -open set in $(U,\tau_R(X))$. That is $Ng^*\alpha int(f^{-1}(Nint(B))$. Also for $B\subseteq V$, $Nint(B)\subseteq B$ always. Then $f^{-1}(Nint(B))\subseteq f^{-1}(B)$. Tharefore $Ng^*\alpha int(f^{-1}(Nint(B)))\subseteq Ng^*\alpha int(f^{-1}(B))$. That is $f^{-1}(B)\subseteq Ng^*\alpha int(f^{-1}(B))$. Also $Ng^*\alpha int(f^{-1}(B))\subseteq f^{-1}(B)$. Hence $f^{-1}(B) = Ng^*\alpha\ int(f^{-1}(B))$ which implies that $f^{-1}(B)$ is $Ng^*\alpha$- open in $(U,\tau_R(X))$ for every nano open set B of . Therefore $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ –continuous.
Q

Example 3.21. From the example (3.8), Let us define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=z$, $f(b)=x$, $f(c)=w$ and $f(d)=y$, Here f is $Ng^*\alpha$ -continuous. Since the inverse image of every nano open set in $(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ - open in $(U,\tau_R(X))$. Let $B = \{z\}\subset V$. Then $Ncl(B) = \{x, z, w\}$. Hence $f^{-1}(Ncl(B)) = f^{-1}(x, z, w) = \{a, b, c\}$. Also $f^{-1}(z) = \{a\}$. Hence $Ng^*\alpha cl(f^{-1}(B)) = Ng^*\alpha cl(a) =\{a\}$. Thus $Ng^*\alpha cl(f^{-1}(B))\neq f^{-1}(Ncl(B))$. Also when $A = \{x, z, w\}\subseteq V$, $f^{-1}(Nint(A)) = f^{-1}(x, w) = \{b, c\}$. But $Ng^*\alpha int(f^{-1}(A)) = Ng^*\alpha int(\{x, w\}) = \{b, c\}$. But $Ng^*\alpha int(f^{-1}(A)) = Ng^*\alpha int(\{x, z, w\}) = \{a, b, c\}$. That is $f^{-1}(Nint(A))\neq Ng^*\alpha int(f^{-1}((A))$. Thus the equality does not hold in the above theorem.

4. Nano $g^*\alpha$ -irresolute functions

In this section we define the class of $Ng^*\alpha$ -irresolute functions which is included in the class of $Ng^*\alpha$ -continuous functions. Also we study some of their basic properties.

Definition 4.1. Let $(U, \tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is Nano $g^*\alpha$ -irresolute(briefly $Ng^*\alpha$ -irresolute) on U if the inverse image of every Nano $g^*\alpha$ -open set in V is Nano $g^*\alpha$ -open in U.

Theorem 4.2. A function $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute, then f is $Ng^*\alpha$-continuous.

Proof. Let $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute, then the inverse image of every $Ng^*\alpha$ -closed set in $(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -closed in $(U, \tau_R(X))$. Let F be nano closed in $(V,\tau_{R'}(Y))$, then f is $Ng^*\alpha$ -closed in $(V,\tau_{R'}(Y))$ and f is $Ng^*\alpha$ -irresolute. Hence $f^{-1}(F)$ is $Ng^*\alpha$ -closed. Therefore f is $Ng^*\alpha$ -continuous.
Q

Remark 4.3. The converse of the above theorem need not be true as seen from the following example.

Example 4.4. Let Let $U = \{a, b, c, d\}$, $X = \{b, d\}$ and $U\backslash R = \{\{b\},\{c\},\{a, d\}\}$. Then $\tau_R(X) = \{U, \phi, \{b\}, \{a, d\}, \{a, b, d\}\}$, and $\tau_R(X)^c = \{U, \phi, \{a, c, d\}, \{b, c\}, \{c\}\}$. Also $V = \{x, y, z, w\}$, $Y = \{x, y\}$, and $V\backslash R' = \{\{x, y\},\{z\},\{w\}\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x, y\}\}$, and $\tau_{R'}(Y)^c = \{V, \phi, \{z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ by $f(a) = y$, $f(b) = x$, $f(c) = z$ and $f(d) = w$. Then $f^{-1}(\{z, w\}) = \{c, d\}$. Thus $\{c, d\}$ are $Ng^*\alpha$ -closed sets in $(U, \tau_R(X))$. That is the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -closed set in $(U, \tau_R(X))$. Therefore f is $Ng^*\alpha$ -continuous. But f is not $Ng^*\alpha$- irresolute, since $f^{-1}(\{w\}) = \{d\}$. which is not $Ng^*\alpha$ -closed in $(U, \tau_R(X))$ where as $\{d\}$ is $Ng^*\alpha$ -closed in $(V,\tau_{R'}(Y))$. Thus a $Ng^*\alpha$ –continuous function is not $Ng^*\alpha$ –irresolute.

Theorem 4.5. A functionand $g : (V, \tau_R(Y)) \to (W,\tau_R(Z))$ are both $Ng^*\alpha$ -irresolute, then $f : (U, \tau_R(X)) \to (V, \tau_R(Y))$ is $Ng^*\alpha$ -irresolute, then $g \circ f : (U, \tau_R(Y)) \to (W, \tau_R(Z))$ is $Ng^*\alpha$ -irresolute.

Proof. Let A be a $Ng^*\alpha$ -open in W. Then $g^{-1}(A)$ is $Ng^*\alpha$ -open in V, since g is $Ng^*\alpha$ - irresolute and $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $Ng^*\alpha$ -irresolute. Hence $g \circ f$ is $Ng^*\alpha$ - irresolute. Q

Theorem 4.6. If $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(z))$ is $Ng^*\alpha$ -continuous, then $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

Theorem 4.7. If $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -continuous and $g : (V,\tau_{R'}(Y)) \to (W, \tau_{R''}(z))$ is nano -continuous, then $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

Theorem 4.8. If $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V,\tau_{R'}(Y)) \to (W, \tau_{R''}(z))$ is nano α -continuous, then $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

(1) If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(z))$ is nano pre- continuous, then $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

(2) If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(z))$ is nano regular- continuous, then $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

(3) If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(z))$ is nano gs- continuous, then $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(z))$ is $Ng^*\alpha$ - continuous.

(4) If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(z))$ is nano sg- continuous, then $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(z))$ is $Ng^*\alpha$ -continuous.

(5) If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -irresolute and $g : : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(z))$ is nano gpr- continuous, then $g \circ f :$ is $Ng^*\alpha$- continuous.

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