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Use of Waiting Time in the$M^X/G(a,b)/1$ Queuing System with Interruptions and Secondary Services

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Abstract- In this paper, we have discussed a method for the management of waiting in $M^X/G(a,b)/1$ system by utilizing distinct waiting times for service arrivals and server breakdowns. With this approach, we are able to process large batches of items in the most efficient manner while also catering to the requirements of large groups of people who arrive at the same time.The model is able to estimate the probability that there will be a large number of people waiting in line at any given time. In addition to this, it offers examples of how the queue size shifts over the course of time.

Keywords: $M^X/G(a,b)/1$ *, setup time, vacation period, busy period and queue size.*

1. Introduction-The holiday shift system provides a method for companies to guarantee that their products and services will continue to be accessible at all times, even if one of their servers is away on vacation. This occurs whenever a server is physically separated from its primary service centre for an undetermined period of time. It is sometimes necessary to perform maintenance on a server when it is busy because the server needs to be taken offline. Because of this, people are unable to use it to complete their tasks.Whenever there are no customers in the store, it is customary for the business to be closed in the primary service centre, one way to make use of a server located in a secondary service centre is to use it.The fact that a server is able to put its idle time to productive use by working on other tasks while it is idle indicates that the queuing model is applicable to some real-world service delivery systems. **Jain and Jain (2010)** conducted research into the process by which queues are created in a computer when various components of the computer become inoperable.The researchers **Ke et al. (2011)**looked at a Markovian queuing system in which some of the servers were unreliable for the purpose of this study.T-preemptive priority string is the name given to the newly introduced priority string. This thread is used to assist in getting things done first (such as emergency situations), before moving on to things that may take longer by **Kim in 2012. Chakravarthy in 2014** did research on a model that has a Markov entry process, and it has customers entering in a sequential order. **Choudhury and Ke (2014)** wrote an article in which they instructed the stable mode behaviour of an M/G/1 queue with public re-effort time and Bernoulli's holiday schedule for an unreliable server that includes a period of downtime and a period of delay. They demonstrated the behaviour of an M/G/1 queue under stable mode conditions. **Dimitriou (2015)** did some research on a retry queue, which is a type of queue that can be utilised to model fault-tolerant systems. Using this queue, we are able to determine how to recover from system errors by experimenting with a variety of potential solutions until we find one that is successful. **Singh et al (2016)** did research on the best way to queue packets in situations in which there are a large number of unreliable servers. **Permarathon and others (2017)** proposed a method to manage re-entries of spectrum unused (SU) in a way that takes into account the duration of their occurrence. This method was developed to manage re-entries of spectrum unused in a more efficient manner.

2. Model formulation and explanation: The queuing system for servers is covered in this paper. In order for the system to begin functioning, the queue must grow to a certain size. A queue's length indicates how many people are waiting to be served. Until the queue length is less than, the server will wait. When the server is in passive mode, it can switch between serving different clients at once. We refer to this as "secondary service." When there are too many people waiting, the server stops providing the secondary service and goes back to serving each person in the queue individually.If the queue length is not reached, the unit service is continuously provided for the number of customers specified by 'a– 1'. But in case of server failure, the service does not stop and continues by doing some work for the current batch. Technical arrangements, for example, a soft-flow dyeing device that detects server failures, may be in place to ensure that the process continues even when the server is refreshed.After renewing the server, if it fails with a certain probability, it will be replaced. If the server does not crash, it will continue to provide service until a queue is created or its renewal time expires. Failing that, it serves whenever there is a queue.If there are fewer than one customer in line, the server will leave for maintenance. If there are still fewer items in the queue than a, the server will continue to wait in a state of inactivity (sleep) until it reaches the queue value of a.The aforementioned model has the capability of delivering an estimate of the queue probability at any given point in time. In addition, numerous performance metrics are presented, along with images illustrating the behaviour of queuing systems.

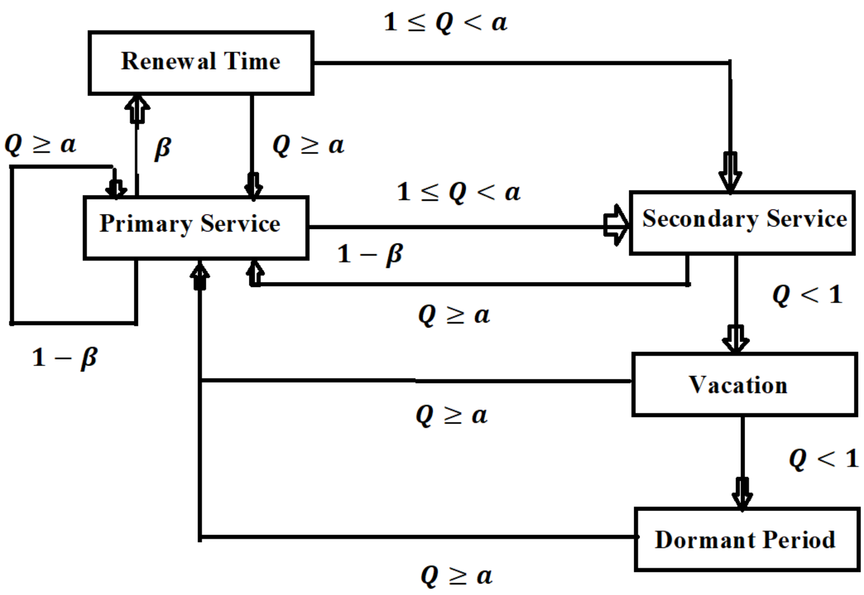

Figure 1: Schematic Representation of the Model

3. Mathematical Model: The Poisson entry rate λ, X be the size of the random entry pool, X, $P(k)$ is the probability that k customers enter the batch, X(z) and the total number of customers in the batch, N_q are all defined below (t). In this case, we'll refer to $N_s(t)$as the number of customers who have been served at time t and $N_w(t)$as the number of customers who are waiting to be served.

4. Analysis of the Model: The Laplace-Stieltjes transform is a function that takes a sequence of numbers and transforms them into a new sequence of numbers that is comparable to, but not exactly the same as, the original sequence of numbers and is defined as

$$M_X(s) = E(e^{-sX}) = \int_0^\infty e^{-sX} f_X(x)\, dx$$

The Laplace-Stieltjes transform is denoted by the notation $M_V(s)$. This notation refers to a function that produces a probability distribution after receiving as an input the amount of time a person anticipates they will spend on vacation.

$$M_V(s) = \frac{\gamma}{s+\gamma}$$

where X and V are independent variables

5. Stationary queue-size distribution:

Let $S(x)$ be the probability distribution function of S.

$V(x)$be the probability distribution function of V.

Furthermore, it is assumed that $V(0) = 0, V(\infty) = 1, S(0) = 0\ \&\ S(\infty) = 1\ and\ V(x) \&\ S(x)$ are continuous at $x = 0$, so that

$$\mu(x)dx = \frac{d\,S(x)}{1-S(x)} \&\ v(x)dx = \frac{d\,V(x)}{1-V(x)}$$

are the first order differential functions of $S\ \&\ V$ respectively.

The number of items in the queue at time t is denoted by$N_q(t)$, and the amount of time that has passed since the beginning of service is denoted by $S^0(t)$. $V^0(t)$represents the amount of time that has been spent in setting as of time t. There are a few uncontrollable elements that contribute to the overall service time at time t.

$$C(t) = \begin{cases} 0, & if\ the\ system\ is\ in\ idle\ state\ at\ time\ t \\ 1, & if\ the\ system\ is\ in\ setup\ state\ at\ time\ t \\ 2, & if\ the\ system\ is\ in\ busy\ state\ at\ time\ t \end{cases}$$

The creation of a bivariate Markov process$\{N_q(t),\ \delta(t)\}$ requires the addition of two new variables: $S^0(t) \& V^0(t)$. These allow us to monitor the progress of the process at any given time more accurately.

where $\delta(t) = 0$ if $C(t) = 0$,B

$\delta(t) = V^0(t)\ if\ C(t) = 1,$

$\delta(t) = S^0(t)\ if\ C(t) = 2$

Define, $R_n(t) = P\big[N_q(t) = n,\ \delta(t) = 0\big]; n = 0, 1, 2, \ldots..$

$P_{0,n}(x,t)\,dx = P\big[N_q(t) = n, \delta(t) = V^0(t);\ \ x < V^0(t) \le x + dx\big];\ \ x > 0, n \ge 1$

$P_{1,n}(x,t)\,dx = P\big[N_q(t) = n, \delta(t) = S^0(t);\ \ x < S^0(t) \le x + dx\big];\ \ x > 0, n \ge 1$

If the system utilization coefficient is equal to or greater than $\rho = \lambda\, E(X)\, E(S)$,then the expected number of arrivals during a period of inactivity and a period of random adjustment is equal to $\theta = [1 + \lambda\, E(X)\, E(V)]$. This indicates that there will not be any changes to the PGF of the queue size distribution.

$$P(z) = \left[\frac{1 - z\, V^*(\lambda - \lambda\, X(z))}{C(\theta)(1 - X(z))}\right]\left[\frac{(1-\rho)(1-z)S^*(\lambda - \lambda\, X(z))}{S^*(\lambda - \lambda\, X(z)) - z}\right]$$

The Laplace-Stilts transform (LST)of V & $S^*(s)$ is applied to the total number of people who have arrived in the past in order to calculate the number ofresiduals, which are new arrivals in the current time interval is given as:

$$\varsigma(z) = \frac{1 - z\,V^*(\lambda - X(z))}{C(\theta)[1 - X(z)]}$$

The probability generating function (PGF) of the queue size distribution is a mathematical equation$M^X/G/1$that determines the frequency with which a particular number of items is chosen. This equation makes use of the following data: the size of the queue (expressed in terms of items), the number of items contained within each bucket, and the total number of items contained within each bin & is given as:

$$P(z, M^X/G/1) = (1 - z)(1 - \rho)\frac{S^*(\lambda - \lambda\,X(z))}{S^*\big(\lambda - \lambda\,X(z)\big) - z}$$

6. Performance measures:

This model estimates the amount of time necessary to finish a predetermined quantity of work by taking into account the current state of the system.

6.1 Expected queue length:

$$E(Q) = \lim_{z \to 1} P'(z)$$

6.2 Expected waiting time in the queue:

$$E(W) = \frac{E(Q)}{\lambda\,E(X)}$$

6.3 Expected queue length of busy period: SET stands for the delay time, and if we assume that B represents the queue occupancy period with a length of $M^X/E_k/1/SET$, then the formula for calculating the expected length of B under steady-state conditions is as follows:

$$E(B) = \frac{E(X)\,E(S)}{1-\rho} + \frac{\rho\,E(X)\,E(V)}{1-\rho}$$

6.4 Expected queue length of idle period:

Based on the random variable "I," this equation gives us information about the average amount of time an individual will be unemployed for.

$$E(I) = \frac{1}{\lambda}\sum_{j=0}^{a-1} Y_j$$

The value of Y_j represents the probability that there will be a particular number of customers waiting in line for the completion of the primary and secondary services.

7. Numerical Simulation:

In this research, we investigate the effect that the arrival rate has on performance metrics by setting $a = 2,\ b = 4,\ \varepsilon = 10,\ \eta = 8,\ \delta = 0.2$respectively.

Table 1: Performance Metrics vs. Time to Arrive

λ	$\mu = 3,\quad \mu' = 2$			
	$E(Q)$	$E(W)$	$E(B)$	$E(I)$
3.0	1.7291	1.1274	2.5432	0.9265
3.3	1.1065	1.1351	2.70962	0.8592

3.6	2.3629	1.1568	2.8383	0.8042
3.9	2.7146	1.1926	3.3742	0.7327
4.2	3.2724	1.2386	3.6160	0.6834
4.5	3.6927	1.2791	3.8641	0.6426

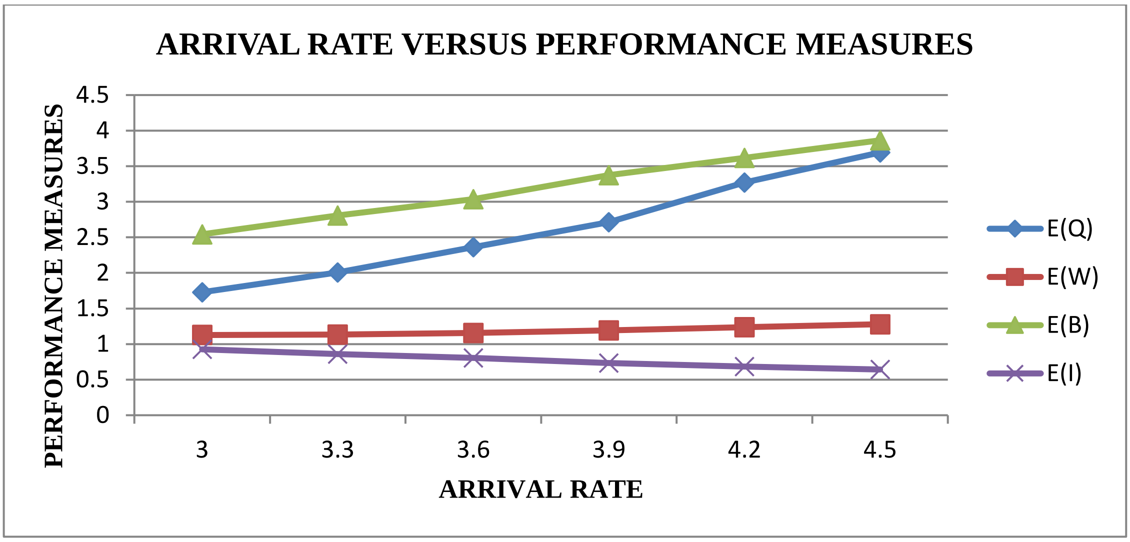

Fig 1: Performance Metrics vs. Time to Arrive

Table 2: Renewal Rates' Impact on Efficiency Metrics

η	$E(Q)$	$E(W)$
3	4.7291	7.0032
3.5	3.1065	6.60962
4	3.3629	5.8383
5	2.7146	5.6742
6	2.5724	5.2160
7	2.2927	4.00641

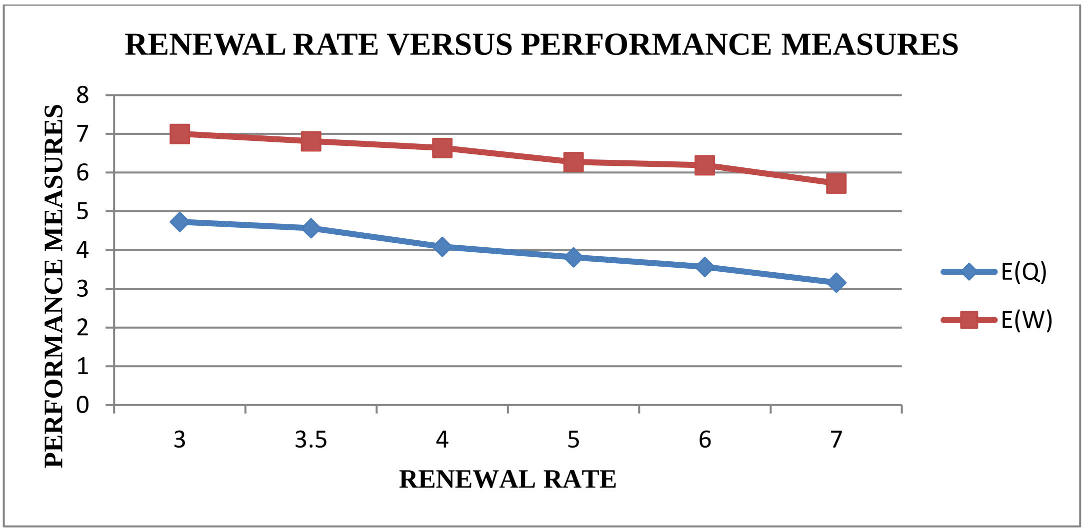

Fig 2: Renewal Rates' Impact on Efficiency Metrics

Table 3: Modifying the Probability of Failure Rates for Evaluation Purposes

δ	$E(Q)$	$E(W)$	$E(B)$	$E(I)$
0.1	3.4361	1.5327	2.4436	0.1292
0.2	4.5468	1.7263	2.4724	0.1145
0.3	6.5982	1.8354	2.5193	0.0945
0.4	7.9430	2.0345	2.5682	0.0826
0.5	9.0341	2.2764	2.6192	0.0523

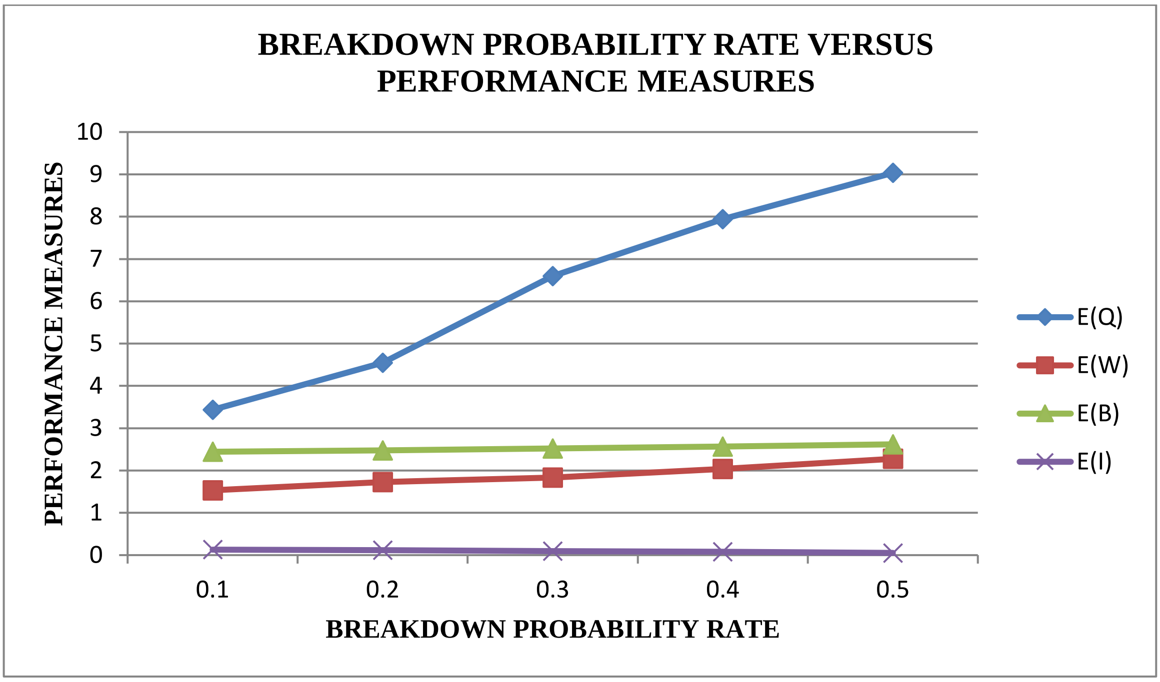

Fig 3: Modifying the Probability of Failure Rates for Evaluation Purposes

8. Concluding Remarks: The graph and table in the numerical section demonstrate that waiting time, busy period length, and idle period length all increase with the number of people arriving at a location. But while waiting times have decreased, idle times have decreased less.The length of time that people wait in line, the amount of time that people are busy, and the amount of time that people are idle all increase as the probability of a server failure rises.

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