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Extension of the - Function to An

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Abstract

The delta function plays a vital role in many areas of mathematics. Our objective in this paper is to extend it to higher dimensional spaces and to study some of its fundamental properties.

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The - function

1.1 Definition:

Let ℝ be the set of real numbers and ℂ be the set of complex numbers.

The - function on a subset E of ℝ or ℂ is the function : E {0, 1} defined by

$$\delta(x) = \begin{cases} 0 \; if \;\; x = 0 \\ 1 \; if \;\; x \neq 0. \end{cases} \quad (1)$$

The first observation we make is that is a mininorm on X = ℝ or ℂ. Let us define a mininorm before developing a proof for this basic observation.

1.2 Definition

Let X ebe ea vector espace eover eK = eℝ or eℂ. A mininorm on eX eis a function w = eX ℝ which satisfy the following conditions:

$$\text{(a) } w(x) \geq 0 \text{ for all } x \in X \quad (2)$$

$$\text{and } w(x) = 0 \text{ if and only if } x = 0 \quad (3)$$

$$\text{(b) } w(\alpha x) = w(x) \text{ for all } x \in X \text{ and } \alpha \in \mathbf{K} \quad 0 \neq \alpha \in \mathbf{K} \quad (4)$$

$$\text{(c) } w(x+y) \leq w(x) + w(y) \text{ for all } x, y \in X \quad (5)$$

A vector space X with a mininorm w defined on it eis ecalled a mininormed space eand is, in general, edenoted eby (eX, w).

Note: Every mininorm w induces a metric dw defined by

$$d_w(x, y) = w(x - y) \text{for all } x, y \in X. \quad (6)$$

1.3 Proposition

Let $X = \square$ or $\square$. Then δ is a mininorm on X.

Proof :

Condition (a) for a mininorm is obvious from the definition of .

To verify (b) $x \in X$ take and $\alpha \in \mathbf{K}$ with $\alpha \neq 0$

If $x = 0$, then $\alpha x = 0$ so that $\delta(x) = \delta(\alpha x) = 0.$

If $x \neq 0$, then $\alpha x \neq 0$ so that $\delta(x) = \delta(\alpha x) = 1.$

Now, we prove (c).

Suppose x + y = 0. Then (c) is obvious.

Now suppose x + y 0. Then, atleast one of x and y is non zero. Without loss of egenerality, we emay eassume that x 0. Then $\delta(x) = 1.$,

$$So,\ \delta(x) + \delta(y) \geq 1. \text{ But } \delta(x + y) = 1$$

$$\text{Hence } \delta(x + y) \leq \delta(x) + \delta(y).$$

1.4 Remark:

It can be easily checked that whenever w is a mininorm on , rw is also a mininorm on where r is any real number ≠ 0. Hence, for any real number r ≠ 0, r□ is also a mininorm on . r□ is the function given by

$$r\delta(x) = \begin{cases} 0 \ if \ \ x = 0 \\ 1 \ if \ \ x \neq 0 \end{cases} \quad (7)$$

we may denote $r\delta$ by δ_r.

2. Extension of the □ - function to .

2.1 Definition:

Let $x = (x_1, x_2, ..., x_n) \in \square^n.$

Define $\delta(x)$ by

$$\delta(x) = (\delta(x_1), \delta(x_2), ..., \delta(x_n)) \quad (8)$$

Let us now define an order relation on .

2.2 Definition:

Let $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n) \in \square^n.$ We say that $x \leq y$ and only if $x_i \leq y_i$ all i = 1, 2, 3, .. n.

It is clear that ≤ is a partial order relation on .

As an illustration, (2, 0, -3) ≤ (5, 1, 0) in $\square^3$.

But the vectors (2, 0, -3) and (5, -1, 0) are not comparable with respect to this order. Thus, the law of trichotomy does not hold in with respect to this order for n > 1.

It is now iinteresting ito inote that most of ithe iproperties δ □ on hold for the extended □ on .

2.3 Proposition: δ in satisfies the following:

(a) $\delta(x) \geq 0$ for all $x \in \mathbb{R}^n$ and $\delta(x) = 0$ if and only if $x = 0$.

(b) $\delta(rx) = \delta(x)$ for all $x \in \mathbb{R}^n$ and for all $0 \neq r \in \mathbb{R}$.

(c) $\delta(x+y) \leq \delta(x) + \delta(y)$ for all $x, y \in \mathbb{R}^n$.

Proof:

Let $x = (x_1, x_2, ..., x_n)$ *and* $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n$.

Then, clearly $\delta(x) = (\delta(x_1), \delta(x_2), ..., \delta(x_n)) \geq 0,$

since $\delta(x_i) \geq 0$ *for all i.*

And $\delta(x) = 0$ if and only if $\delta(x_i) = 0$ for all i

if and only if $x_i = 0$ for all i if and only if x = 0.

For r ≠ 0, consider

$$\begin{aligned}\delta(rx) &= \delta(rx_1, rx_2, ..., rx_n)\\ &= (\delta(rx_1), \delta(rx_2), ..., \delta(rx_n)\\ &= (\delta(x_1), \delta(x_2), ..., \delta(x_n))\\ &= \delta(x).\end{aligned}$$

Further,

$$\delta(x+y) = \delta(x_1+y_1, x_2+y_2, ..., x_n+y_n)$$

$$= (\delta(x_1+y_1), \delta(x_2+y_2), ..., \delta(x_n+y_n)) \qquad (9)$$

Now, $\delta(x_i + y_i) \leq \delta(x_i) + \delta(y_i)$ *for all i.*

So, (9) gives

$$\begin{aligned}\delta(x+y) &= (\delta(x_1)+\delta(y_1), \delta(x_2)+\delta(y_2), ..., \delta(x_n)+\delta(y_n))\\ &= (\delta(x_1), \delta(x_2), ..., \delta(x_n)) + (\delta(y_1), \delta(y_2), ..., \delta(y_n))\\ &= \delta(x) + \delta(y)\end{aligned}$$

□

For an x = (x1, x2, …, xn) in , its norm $\|x\|$ is defined by

$$\|x\| = \left(x_1^2 + x_2^2 + ... + x_n^2\right)^{1/2} \qquad (10)$$

For x = (x1, x2, …, xn), put . $|x| = (|x_1|, |x_2|, ..., |x_n|)$ (11)

Also, for . $r \in \mathbb{R}$, *put* $\bar{r} = (r, r, ..., r) \in \mathbb{R}^n$ (12)

For example, = $\bar{1}$ = (1, 1, …, 1).

Now we have the result:

2.4 Proposition:

Let . $x \in \square^n$

(a) If $x \geq 1,$ then $\|x\| \geq \|\delta(x)\|$

(b) If . $|x| \leq 1,$ then $\|x\| \leq \|\delta(x)\|$

Proof:

Suppose $x \geq 1.$ That is, $x_i \geq 1$ for all $i.$

So, $x_i^2 \geq 1^2 = \delta(x_i)^2$ for all i.

Hence, $x_1^2 + x_2^2 + ... + x_n^2 \geq \delta(x_i)^2 + \delta(x_2)^2 + ... + \delta(x_n)^2$,

which implies

$\|x\| \geq \|\delta(x)\|.$

Now, if $|x| \leq 1,$ then $|x_i| \leq 1$ for all $i,$

so that $x_i^2 \leq 1^2 = \delta(x_i)^2.$

Hence we get, $\|x\| \leq \|\delta(x)\|.$ □

Note:

It is not true $x \leq 1$ that implies $\|x\| \leq \|\delta(x)\|.$

For example, let x = (-2, 0, 0) $\in \square^3.$

Then . $x \leq 1$

But $\|x\| = 2 \geq \|\delta(x)\|.$

2.5 Definition:

For i =1, 2, .., n, ei is the vector defined by

ei = (0, 0, …, 1, 0,…,0), with 1 occurs in the ith place and all other co ordinators are 0.

Remark:

$$\delta(e_i) = e_i.$$

More generally, for r ≠ 0

$$\begin{aligned}\delta(re_i) &= \delta(0, 0, ..., r, 0, 0, ..., 0)\\ &= (0, 0, , ..., \delta(r), 0, 0, ..., 0)\\ &= (0, 0, ..., 1, 0, 0, ..., 0)\\ &= e_{i.}\end{aligned}$$

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