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Research Article

SOME CHARACTERIZATION OF SFG-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we have introduced and studied the topological properties of SFG-closure, SFG-interior and SFG-interior points, by using the concept of Semi Feebly Generalized open sets and Semi Feebly Generalised closed set (SFG-closed sets), A subset A of a topological space (X,τ) is called SFG-cosed if U contains Feebly closure of A whenever U contains A and U is sg-open in (X,τ). Some interesting results that shows the relationships between these concepts are brought about.

KEYWORDS: SFG-closure, SFG-interior, SFG-closed sets, SFG-open, Feeblyclosure.

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I.INTRODUCTION

In 1970, for the first time the concept of generalised closed set was considered by Levine[3]. After the works of Levine on semi-open sets, various mathematicians turned their attention to the generalisations of topology by considering semi open sets instead of open sets. Maheswari and Jain[4] introduced feebly closed sets. In 1987, P.Bhattacharya and B. K. Lahiri[2] introduced Semi-generalised closed sets in Topological spaces. We[1] have alreadyintroduced a class of generalised closed set SFG-closed set using feebly closed sets and semi generalised open sets.

In this paper we have used the notation of SFG-closed sets and SFG-open sets, we introduce and study the topological properties of SFG-Interior, SFG-Closure of a set, SFG-interior points of a set and show that some of their properties are analogous to those for open sets.

II. PRELIMINARIES

Throughout this paper, (X,τ) (simply X) always mean topological space on which no separation axiom is assumed unless otherwise mentioned.(X,τ)will be replaced X if there is no change of confusion. For a subset A of a topological space X, $cl(A)$, $int(A)$, denote the closure of A, interior of A respectively.

We recall some of the definitions and results which are used in the sequel.

Definition 2.1

A subset A of a topological space (X,τ) is called

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1. SFG-closed set [1], if $fcl \subseteq U$ whenever $A \subseteq U$ and U is semi generalised open(sg-open) set in X. The family of SFG-closed set is denoted by SFG-C(X).
2. SFG-open set if $X \setminus A$ is SFG-closed set int$A$. The family of SFG-open set is denoted by SFG-O(X).

Definition 2.2

A subset A of a topological space (X, τ) and $A \subseteq X$ is called

1. The closure of A, denoted by $cl\ (A)$ and is defined by the intersection of all closed set containing A.
2. The interior of A, denoted by $int(A)$ and is defined by the union of all open sets contained in A.

III. SEMIFEEBLY GENERALISED INTERRIOR OPERATOR

Definition 3.1

Let A be a subset of a topological space (X, τ). Then the union of all SFG-open sets contained in A is called the SFG-interior of A and it is denoted by $int_{SFG}(A)$.

That is, $int_{SFG}(A) = \cup \{V : V \subseteq A$ and $V \in$SFG-O(X)$\}$

Definition 3.2

Let A be a subset of a topological space X. A point $p \in X$ is called a SFG-interior point of A if there exists a SFG-open set G such that $p \in G \subseteq A$

Proposition 3.3

For any $A \subseteq X, int(A) \subseteq int_{SFG}(A)$

Proof:

Straight forward.

Proposition 3.4

For any two subsets $A_1\ and\ A_2$of X.

1. If $A_1 \subseteq A_2$, then $int_{SFG}(A_1) \subseteq int_{SFG}(A_2)$
2. $int_{SFG}(A_1 \cup A_2) \supseteq int_{SFG}(A_1) \cup int_{SFG}(A_2)$

Remark 3.5

Since the union of SFG-open subsets of X is SFG-open in $X, int_{SFG}(A)$ is SFG-open in X.

Proposition 3.6

Let A be a subset of a topological space (X, τ). Then

1. $int_{SFG}(A)$ is the largest SFG-open set contained in A.

2. A is SFG-open iff $int_{SFG}(A) = A$.
3. $int_{SFG}(A)$ is the set off all SFG-interior points of A.
4. A isSFG-open iff every point of A is SFG-interior point of A.

Proof:

1. Being the union of all SFG-open sets, $int_{SFG}(A)$ is SFG-open and contains every SFG-open subset of A. Hence $int_{SFG}(A)$ is the largest SFG-open set contained in A.
2. **Necessity:** Suppose A is SFG-open. Then by Definition of SFG-interior, $A \subseteq int_{SFG}(A)$. But $int_{SFG}(A) \subseteq A$ and therefore $int_{SFG}(A) = A$.
   Sufficiency: Suppose $int_{SFG}(A) = A$. Then by remark 3.5, $int_{SFG}(A)$is SFG-open set. Hence A is SFG-open.
3. $p \in int_{SFG}(A) \Leftrightarrow$ there exists a SFG-open set G such that $p \in G \subseteq A$
   $\Leftrightarrow$ p is a SFG-interior point of A.
   Hence $int_{SFG}(A)$ is the set of all SFG-interior points of A.
4. Follows from (1) and (2).

Proposition 3.7

Let A and B be subsets of (X, τ). Then the following results hold.

a) $int_{SFG}(\emptyset) = \emptyset$ and $int_{SFG}(X) = X$.
b) If B is any SFG-open set containe in A, then $B \subseteq int_{SFG}(A)$.
c) If $A \subseteq B$, then $int_{SFG}(A) \subseteq int_{SFG}(B)$.
d) int(A)⊆S-$int(A)$⊆ $int_{SFG}(A)$ ⊆sg-$int(A) \subseteq A$.
e) $int_{SFG}\left(int_{SFG}(A)\right) = int_{SFG}(A)$

Proof

a) Since $\emptyset$ is the only SFG-open set contained in $\emptyset$, then $int_{SFG}(\emptyset) = \emptyset$. Since X is SFG-open and$int_{SFG}(X)$is the union of all SFG-open sets contained in X, $int_{SFG}(X) = X$.
b) Suppose B is SFG-open set contained in A. Since $int_{SFG}(A)$ is the union of all SFG-open set contained in A, then we have $B \subseteq int_{SFG}(A)$.
c) Suppose $A \subseteq B$. Let $p \in int_{SFG}(A)$. Then p is a SFG-interior point of A and hence there exists a SFG-open set G such that $p \in G \subseteq A$. Since $A \subseteq B$, then $p \in G \subseteq B$. Therefore p is a SFG-interior point of B. Hence $p \in int_{SFG}(B)$.
d) Since every SFG-open set is sg-open, $int_{SFG}(A)$ ⊆sg-int(A). Since every s-open set is SFG-open, s-int(A) ⊆sg-int(A). Eery open set s-open, $int(A)$⊆s-$int(A)$.
   Therefore int(A)⊆s-int(A) ⊆ $int_{SFG}(A)$ ⊆sg-int(A) ⊆ A.
e) By remark 3.5, $int_{SFG}(A)$ is SFG-open and by Prop 3.6(2), $int_{SFG}\left(int_{SFG}(A)\right) = int_{SFG}(A)$.

Proposition 3.8

Let A and B are subsets of a topological space X. Then

i) $int_{SFG}(A) \cup int_{SFG}(B) \subseteq int_{SFG}(A \cup B)$
ii) $int_{SFG}(A \cap B) \subseteq int_{SFG}(A) \cap int_{SFG}(B)$

Proof

Let A and B be subsets of X.

i) By Prop (3.7)(c), $int_{SFG}(A) \subseteq int_{SFG}(A \cup B)$ and $int_{SFG}(B) \subseteq int_{SFG}(A \cup B)$. Which implies, $int_{SFG}(A) \cup int_{SFG}(B) \subseteq int_{SFG}(A \cup B)$.

ii) Again, by Prob(3.7)(c), $int_{SFG}(A \cap B) \subseteq int_{SFG}(A)$ and $int_{SFG}(A \cap B) \subseteq int_{SFG}(B)$. Which implies, $int_{SFG}(A \cap B) \subseteq int_{SFG}(A) \cap int_{SFG}(B)$.

Proposition 3.9

For any subset A of X,

i) $int\left(int_{SFG}(A)\right) = int(A)$

ii) $int_{SFG}\left(int(A)\right) = int(A)$

Proof

i) Since $int_{SFG}(A) \subseteq A, int\left(int_{SFG}(A)\right) \subseteq int(A)$. By Prop(3.6)(4), $int(A) \subseteq int(int_{SFG}(A))$ and $int(A) = int(int(A)) \subseteq int(int_{SFG}(A))$.
Hence $int(int_{SFG}(A))=int(A)$

ii) Since $int(A)$ is open and hence SFG-open, by Prob(3.5)(2),$int_{SFG}\left(int(A)\right) = int(A)$.

IV. SEMI FEEBLY GENERALIZED CLOSURE OPERATOR

Definition 4.1

Let A be a subset of a topological spaces. Then the intersection of all SFG-closed sets in X containing A is called the SFG-closure of A and it is denoted by $cl_{SFG}(A)$.

That is, $cl_{SFG}(A) = \cap \{E: A \subseteq E \; and \; E \in SFG - C(X)\}$

Remark 4.2

Since the intersection of SFG-closed set is SFG-closed, $cl_{SFG}(A)$ is SFG-closed.

Proposition 4.3

Let A be a subset of a topological space (X, τ), then

i) $cl_{SFG}(A)$ is the smallest closed SFG-closed set containing A.

ii) A is SFG-closed iffcl_{SFG}.

Proposition 4.4

Let A and B be two subsets of a topological space X, Then the following result holds,

a) $cl_{SFG}(\emptyset) = \emptyset$ and $cl_{SFG}(X) = X$

b) If B is any SFG-closed set containing A, then $cl_{SFG}(A) \subseteq B$

c) If $A \subseteq B$, then $cl_{SFG}(A) \subseteq cl_{SFG}(B)$.

d) $A \subseteq sg - cl(A) \subseteq cl_{SFG}(A) \subseteq scl(A) \subseteq cl(A)$

e) $cl_{SFG}(cl_{SFG}(A)) = cl_{SFG}(A)$

Proposition 4.5

Let A and B be two subsets of a topological space X. Then

i) $cl_{SFG}(A) \cup cl_{SFG} \subseteq cl_{SFG}(A \cup B)$
ii) $cl_{SFG}(A \cap B) \subseteq cl_{SFG}(A) \cap cl_{SFG}(B)$

Proposition 4.6

For a subset A of X and $p \in X, p \in cl_{SFG}(A)$ if and only if $V \cap A \neq \emptyset$ for every SFG-open set V containing p.

Proof

Necessity: Suppose $p \in cl_{SFG}(A)$. If there is a SFG-open V containing p such that $V \cap A = \emptyset$, then $A \subseteq X \backslash V$ and $X \backslash V$ is SFG-closed and hence $cl_{SFG}(A) \subseteq X \backslash V$. Since $p \in cl_{SFG}(A)$, then $p \in X \backslash V$ which contradicts to $p \in V$.

Sufficiency: Assume that $V \cap A \neq \emptyset$ for every SFG-open set V containing p. If $p \notin cl_{SFG}(A)$, then there exists a SFG-closed set E such that $A \subseteq E$ and $p \notin E$. Therefore $p \in X \backslash E, A \cap (X \backslash E) = \emptyset$ and $X \backslash E$ is SFG-open. This is a contradiction to our assumption. Hence $p \in cl_{SFG}(A)$.

Proposition 4.7

For any subset A of X,

a) $cl(cl_{SFG}A)) = cl(A)$
b) $cl_{SFG}(cl(A)) = cl(A)$

RELATION BETWEEN SFG-CLOSURE AND SFG-INTERIOR

Proposition 4.8

Let A be a subset of a space X. Then the following are true:

i) $\left(int_{SFG}(A)\right)^c = cl_{SFG}(A^c)$
ii) $int_{SFG}(A) = (cl_{SFG}(A^c))^c$
iii) $cl_{SFG}(A) = (int_{SFG}(A^c))^c$

Proof:

i) Let $p \in (int_{SFG}(A))^c$. Then $\notin int_{SFG}(A)$. That is, Every SFG-open set V containing p is such that $V \notin A$. ThusSFG-open set V containing $p\ is\ such\ that, \cap A^c \neq \emptyset$. By Prob(4.6), $p \in cl_{SFG}(A^c)$ and therefore$(int_{SFG}(A))^c \subset cl_{SFG}(A^c)$.
Conversely, Let $p \in cl_{SFG}(A^c)$. Then again by Prob(4.6) and Definition 3.1, $p \notin int_{SFG}(A)$.Hence $p \in (int_{SFG}(A))^c$. And so $(int_{SFG}(A))^c \supset cl_{SFG}(A^c)$.
Hence $(int_{SFG}(A))^c = cl_{SFG}(A^c)$.
ii) Follows by taking complements in (i).
iii) Follows by replacing A by A^c in (i).

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