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Construction of Minimum Connected Dominating Set by using AJK Algorithm

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Abstract

Dominating set, minimum dominating set and minimum connected dominating set is correct sequence need to study in any wireless network system. One must need to have MCDS for efficient and cost effective wireless network system. A graph G = (V; E), a dominating set D is a subset of V such that any vertex not in D is adjacent to at least one vertex in D. In a graph G = (V; E) a connected dominating set is a subset of vertices such that every vertex is either in the subset or adjacent to a vertex in the subset and the subgraph induced by the subset is connected. A minimum connected dominating set is such a vertex subset with minimum cardinality. In this paper, we present a new method
i.e. AJK algorithm we can easily find a minimum connected dominating set. We can easily see that this algorithm gives a constant performance guarantee. The results show that, despite its simplicity, the proposed algorithm gives very good solutions.

MSC: 05C69

Key Words: AJK Algorithm, Graph, Dominating set, Minimum dominating set; Minimum connected dominating set, wireless networks.

Introduction:

Minimum connected domination is widely used in wireless networks and wireless networks are used in many fields [10] such as environment monitoring, disaster forecast, battlefield detection, traffic control, and disease diagnosis etc. In recent years, CDS as the virtual backbone network in wireless ad hoc and wireless sensor networks (WSNs) has attracted more and more attention of researchers, since the CDS can prolong the lifetime of the wireless network, optimize the energy consumption, transfer data fast, and so forth. Further, how to minimize the size of the CDS is a significant research direction In wireless networks, each node can only communicate with neighbours and its computing capacity is finite. Therefore, the nodes need to cooperate with each other to complete an overall task. If a node wants to transfer information to other nodes outside of the transmitting range (means the destination has multiple hops away from it), a backbone network is needed to transmit the messages effectively. Once the backbone network is constructed, all nodes can communicate with other nonneighbour nodes through intermediate nodes in the backbone network.

Therefore, the nodes that are not in the backbone network can sleep intermittently or shut off for saving energy consumption. It was verified that CDS is an optimal selection as a backbone network for transferring messages [11, 8].

In wireless networks, CDS as a virtual backbone network was first proposed [1]. Beyond that, CDS has many practical applications such as topology control [9], routing [13], and broadcasting [16]. In addition to the size of CDS, our goal is to design an optimization algorithm for considering other algorithm performance indexes such as CDS size, throughput size, time complexity, and message complexity.

In this paper, assume that the AJK algorithm is executed in wireless network and the nodes are deployed on a two dimensional plane. The topology of the network is modelled with a unit disk graph

(UDG) G = (V, E), where V represents the set of nodes in the network and E represents the collection of the network link. And assume that each node v ∈ V has a unique id and its maximum transmission range is R. In addition, if node u is in the maximum transmission range of node v, we think that there is an undirected link between u and v, which is the undirected edge between u and v on graph G. Let (v) be the neighbour set of node v and let dV = |(v)| be the degree of node v. And let Δ = max{dV | v ∈ V} be the maximum degree of the graph. Let
G = (V, E) be the original network topology and DS V_1 is a subset of set V in graph G. For arbitrary node v ∈ V, if node v is not in set V_1, there is at least a neighbour node of v in set V_1. For any two nodes u, v ∈ V_1, if there exists a path that can make them connected by other intermediate nodes which belongs to set V_1, and then we denote set V_1 as CDS. We just need to store the routing information into the nodes in the CDS, which can route messages as soon as quickly. It is clear that the smaller the size of the CDS is, the faster the messages are transmitted and the less the energy consumption of the node is. Therefore, it is important to construct the minimum CDS (MCDS). Unfortunately, it was proved that the MCDS problem was NP-complete [8]. Hence, most of researchers have been devoted to designing approximate algorithm for constructing the MCDS.

In this paper, we propose AJK algorithm, this algorithm works in any graph which is connected. First we find the centre of the given graph then by applying the proposed algorithm we can get a CDS and actually this one is MCDS.

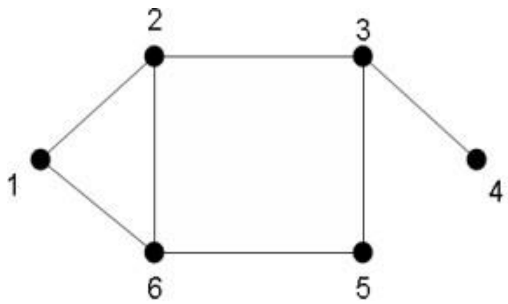
Some Definition:

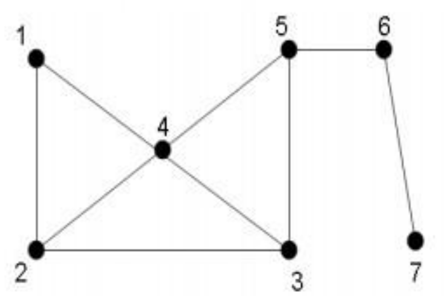
In order to develop the algorithm, we state some definition and introduce some terminology relevant to the paper.

- **Dominating Set** – Dominating Set for a graph G = (V,E) is a subset D of the Vertex Set V such that each vertex u∈V is either in D or adjacent to some vertex v in D. The elements of dominating set are called dominators. Examples of dominating set in a graph G are given below:

Figure 1: {1, 3}, {2, 3, 5} and {1, 2, 3, 4} are Dominating Sets

Figure 2: {4, 6}, {1, 5, 7} and {4, 5, 6} are Dominating Sets





- **Connected Dominating Set** – A Connected Dominating Set (CDS) of a graph G =(V,E) is a set of vertices with two properties:

1. D is a dominating set in G.
2. D induces a connected subgraph of G.

In Fig.1, {2, 3, 5} and {1, 2, 3, 4} are Connected Dominating Sets. Similarly in Fig. 2, {4, 5, 6} is a Connected Dominating Set.

(iii) **Minimum Connected Dominating Set** – A minimum Connected Dominating Set (MCDS) is a connected dominating set with smallest possible cardinality among all the CDSs of G. As in Figs. 1 and 2, {2, 3, 5} and {4, 5, 6} are Minimum Connected Dominating Sets respectively.

(iv) **Independent Set** – Independent Set of a graph G is a subset of the set of vertices such that no two vertices are adjacent in the subset. For example in Fig.1 {1, 3}, {1, 4, 5}, {2, 4, 5} are independent sets.

(v) **Maximal Independent Set** – Maximal independent set (MIS) is an independent set, which is not a subset of any other independent set. i.e. it is a set S such that every edge of the graph has atleast one end point not in S and every vertex not in S has atleast one neighbour in S. An MIS is also a dominating set. Six different Maximal Independent Set of following cubic graph are {1,6}, {2,5}, {3, 8},{4, 7}, {1, 5, 7} and {4, 6, 8, 2}.

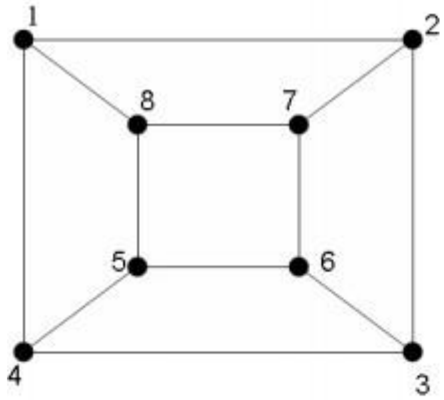


Figure 3: MIS in Cubic Graph

(vi) **Convex hull** – the convex hull for a set of points X in real vector space is the minimum convex set containing X. it is also called convex envelop and denoted by CH(X). It is represented by a sequence of the vertices of the line segment forming the boundary of the convex polygon. As in the following example convex hull of the set {1, 2, 3, 4, 5, 6, 7} of points is shown.

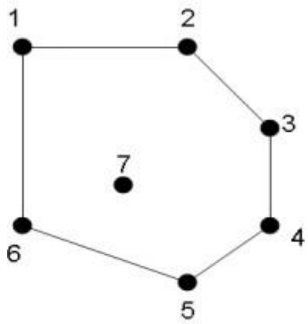


Figure 4: CH ({1, 2, 3, 4, 5, 6, 7})

(vii) **Unit Disk Graph** – A graph G is a Unit Disk graph if there is an assignment of unit disks centered at its vertices such two vertices are adjacent if and only if one vertex is within the unit disk centered at the other vertex.

(viii) **Neighbourhood of a vertex** – Neighbourhoods of a vertex u in a graph G = (V,E) is a set of vertices which are adjacent to u in G. It is denoted by N(u) or

(ix) N[u]. If neighbourhood does not include u itself, then it is called *open neighbourhood* of u and denoted by N(u). As in figure 1, N(1) is {2, 6}, N(2) is {1, 3, 6}, N(3) is {2, 4, 5} and soon. If (x) neighbourhood includes u itself, then it is called *closed neighbourhood* of u and denoted by N[u]. For example in Fig. 2, N[1] is {1, 2, 4} and N[2] is {1, 2, 3, 4}and so on.

(xi) **Distance in graph -** In a connected graph G, the distance d(vi , vj) between two of its vertices v_i

and v_j is the length of the shortest path.

(vi) **Eccentricity and center -** The eccentricity E(v) of a vertex v in a graph G is the distance from v to the vertex farthest from v in G; that is,

$$E(v) = \max_{v_i \in G} d(v, v_i)$$

A vertex with minimum eccentricity in graph G is called a centre of G

Related Work:

In this section, we know a rooted spanning tree T of G constructed from a connected dominating set (CDS). This tree will be used in the routings of all the four group communications. Depending on the type of the group communications, the root of T, denoted by s, is chosen as follows. For broadcast, s is the source of the broadcast; for aggregation or gathering, s is the sink node; for gossiping, s is a graph centre of G. In either case, we use L to denote the graph radius of G with respect to s.

It begins with the construction of a small, short, and sparse CDS of G. We first select a maximal independent set (MIS) I of G in the first-fit manner in a breadth first- search (BFS) ordering (with respect to s) of V. All nodes in I forma dominating set, and hence are referred to as dominators. Then, we select a set C of connectors to interconnect I as follows. Let G' be the graph on I in which there is edge between two dominators if and only if they have a common neighbour. The radius of G' with respect to s is denoted by L'. Clearly, $L' \leq L-1$. For each $0 \leq l \leq L'$, let I_l be the set of dominators of depth l in G'. Then, $I_0 = \{s\}$. For each $0 \leq l < L'$, let P_l be the set of nodes adjacent to at least one node in I_l and at least one node in I_{l+1}, and compute

a minimal cover $C_l \subseteq P_l$ of I_{l+1} (see an illustration in Figure 5). Set *C* □

∪ *L* '□1 *l*= C_l. Then, I∪

C is a CDS of G.

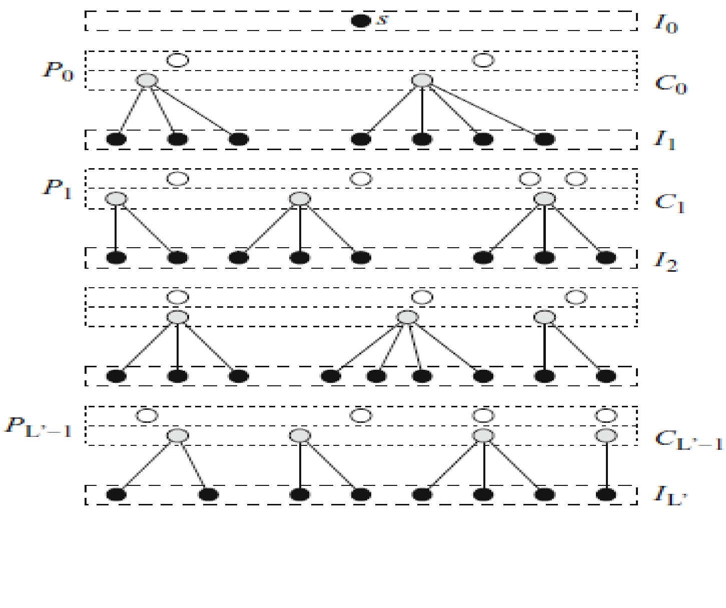


Figure 5: The selection of connectors (marked by gray)

In order to minimize the size of CDS, we devoted to designing approximate algorithm to obtain CDS by the approximation factor |opt|, which is as little as possible. In many cases, however, we need to consider the other performance parameters such as the diameter of CDS, communication consumption of nodes.

As a critical performance index, the size of CDS has attracted more attention of researchers. For constructing CDS as virtual backbone networks in wireless network, Guha and Khuller [5] proposed two heuristic centralized algorithms. The fist algorithm firstly put up all nodes are transformed into white. And the white node with the maximum number of neighbors is selected to be colored black, and all its white neighbors are marked gray. Finally, they formed CDS by all black nodes. The approximation ratio of the algorithm is $2(1+(\Delta))$. The second algorithm is divided into two phases. The first stage is to delete redundant nodes of graph as many as possible. The second phase is to build a Steiner tree for constructing CDS.

And it was proved that the approximation ratio is $\ln \Delta + 3$. Afterwards, two distributed versions of algorithms in [3, 4] were proposed by Das et al. Then Wu and Li proposed a distributed

algorithm [6] to construct a CDS based on two-hop information of neighbours. For arbitrary node in the network, it is added to CDS if there are two nonadjacent neighbours, and all redundant nodes are moved from the network. When topology of network contains complete subgraphs, however, the algorithm cannot execute successfully. In [7], Wan et al. proved that the approximation ratio of [6] is (n), and they proved that the approximation ratio of any MIS is $4|opt| + 1$ in the unit disk graph (i.e., $MIS(G) = 4|opt| + 1$), where opt is the optimal set of CDS. Afterwards, they proposed a distributed algorithm to construct CDS in unit disk graph. In the first phase, they constructed a spanning tree and obtained MIS with the method of searching nodes layer by layer. Later, they constructed a dominating tree whose internal nodes can consist of CDS with performance ratio $8|opt|-2$ and the time complexity and message complexity of O(n), respectively, where opt is the size of optimal solution of CDS and n is the number of nodes in the network. In [14], Funke et al. Proved that the approximation ratio of MIS is $3.453|opt| + 8.292$ by analyzing the coverage area of nodes, and they got CDS with the approximation ratio of 6.9. In [12] and [19], Li et al. and Das et al., respectively, proposed different distributed algorithm of approximation ratio $(4.8 + \ln 5)|opt| + 1.2$. All of these algorithms first construct MIS and then obtain CDS by connecting the MIS nodes through some connectors.

Except the size of CDS, communication consumption of nodes is also an important performance index. It not only involves the energy consumption but also contains the time consumption in communication cost, which are related to the number of hops. Then reducing the number of hops is the most effective way to increase the transmission range. However, the battery power of nodes in backbone requires higher volume. Therefore more and more researchers have been devoted to studying the routing efficient CDS [17–20]. At the same time, the works about minimizing the diameter of CDS are gradually being taken seriously. An algorithm [15] is proposed for constructing CDS with the small diameter, which guaranteed that not only is the diameter of acquired CDS limited to $3D^* + 7$ (where D^* is the diameter of the MCDS), but also the size of CDS is limited to $11.4|opt| + 1.6$. Additionally, some other research facets such as k-connected m-dominating set [24], weakly-dominating set [22], and

k-hop [17] have also attracted much attention by researchers.

Process of AJK Algorithm:

Let us consider a connected graph G, first of all find the centre of a graph and then we start our procedure consider all vertices white now colour black the centre of graph and associated all

vertices Gray then select a gray vertices associated with maximum white vertices make it black and repeat this process till all the vertices are black and gray no white vertices remain This suggests in AJK algorithm:

AJK algorithm. For a given connected graph G, do the following:

Set w := 1; while w = 1 do

if there exists a white or gray vertex such that colouring it in black and its adjacent white vertices in gray would reduce the value of potential function then choose such a vertex to make the value of potential function reduce in a maximum amount else set w := 0;
Clearly, when the while-loop ends, no white vertex will exist, i.e., all black vertices form a dominating set; however, the subgraph induced by black vertices are connected i.e. CDS and actually this one is MCDS.

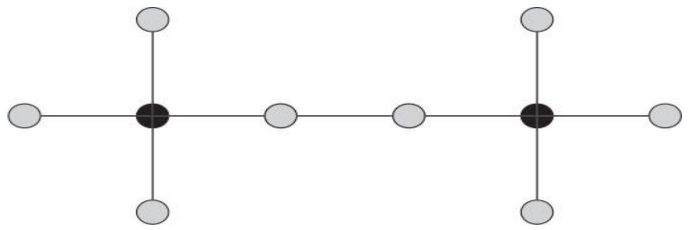


Figure6: Black components

AJK Algorithm to find MCDS:

In this paper, we propose a AJK algorithm. The pseudo code of the algorithm is described as

AJK algorithm for MCDS..

In this paper, three states of a node are represented by colours white, black, and gray. Initially, all nodes are remarked white, also called initial state. A global parameter *count* (its initial value is n, i.e., the number of nodes) is a parameter that is to record the current number of white nodes. We first find the centre of graph. Start with centre of the graph v as the initial node of the CDS and it turns into black and all of its neighbours are coloured gray. Now the value of the parameter *count* substracts 1. And for any node w ∈ (V), node v is removed from set (w); that is, (w) = (w) - v. Then all the nodes in (w) are white. As long as the value of *count* is

greater than 0, each time we select any node v that has the largest number of white vertices associate with gray nodes among all the nodes as a new node in the CDS and it is marked black and all its white neighbours turn into gray. When two or more gray nodes have the same degree then select randomly any vertex but keeping the view that maximum white node to be gray, and continue this process a new element of CDS. The operations are performed iteratively until all nodes are coloured to black and gray. Now the important step is if all black node are connected then it is CDS this CDS may be MCDS this can be easily seen if any black node can be converted in to gray without affected it's CDS property then CDS is MCDS or convert itin to MCDS by making some black node to gray without affected it's CDS property. If required repeat this process till we reach to MCDS. We must remember a important note that while seeing the domination property we must includes all chords and even we consider chords while colouring process that is B/W/G.

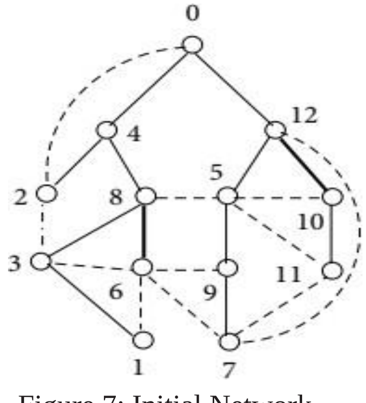


Figure 7: Initial Network

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	-	3	1	2	1	2	3	2	2	3	2	3	1
1	3	-	2	1	3	3	1	2	2	2	4	3	3
2	1	2	-	1	1	3	2	3	2	3	4	4	2
3	2	1	1	-	2	2	1	2	1	2	3	3	3
4	1	3	1	2	-	2	2	3	1	3	3	3	2
5	2	3	3	2	2	-	2	2	1	1	1	1	1
6	3	1	2	1	2	2	-	1	1	1	3	3	2
7	2	2	3	2	3	2	1	-	2	1	2	1	1
8	2	2	2	1	1	1	1	2	-	2	2	2	2
9	3	2	3	2	3	1	1	1	2	-	2	2	2
10	2	4	4	3	3	1	3	2	2	2	-	1	1
11	3	3	4	3	3	1	3	1	2	2	1	-	2
12	1	3	2	3	2	1	2	1	2	2	1	2	-

Table 1: to find eccentricity and centre of graph

E(0) = 3 , E(1) = 4, E(2) = 4, E(3) = 3, E(4) = 3, E(5) = 3, E(6) = 3, E(7) = 3, E(8) = 2

E(9) = 3, E(10) = 4, E(11) = 4, E(12) = 3

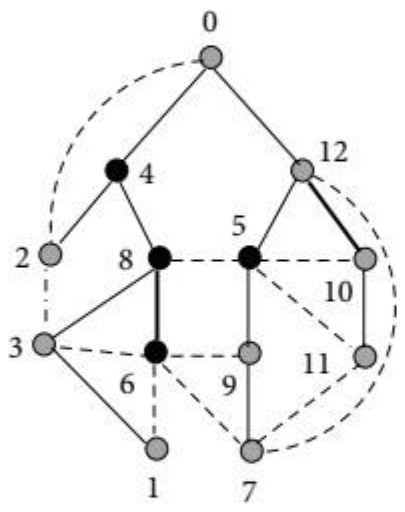


Figure 8: MCDS by AJK algorithm

The result description of constructing operation of CDS is shown in above Figures with executing *AJK* Algorithm. At first, we find a centre of graph. The centre of a tree is 8 which is having the minimum eccentricity, starting from the centre make it black node 8 is colour black say B1 now the connected all vertices colour gray say G1which is 3, 4, 5 & 6. Now select a gray vertex among all gray having maximum associated white vertex this time the vertex is 6 & 5 select any one let us we select 6 make it black and make gray all connected vertex to 6 i.e. 3, 1, 9 and 7 say G2. Now select a gray amongst all G1 & G2 having largest associated white vertex this time the vertex is 5, make it black say B3 and gray to all connected vertex i.e. 10, 11 & 12 which is not yet gray because 9 is already gray and 8 is black, now 4 is the gray vertex having largest associated white vertex amongst all gray make it black and gray to 0 & 2. Now

all the vertices are black and gray no white vertices remain, so we to stop the process and we will see that 4, 5, 6, 8 are connected dominating set and it is MCDS too.

Lemma 1. *CDS is a MCDS in AJK algorithm.*

Proof. Here we prove that set CDS is a MCDS in AJK algorithm. Using the AJK algorithm first we find the centre of the connected graph and then we start the colouring process we can see all the black vertices are connected therefore the result is CDS. Now we will test if removing any black vertex from CDS vanishes its CDS property then this CDS is MCDS or if not then continue this process till CDS property will not change then the latest CDS is MCDS.

AJK Algorithm:

1. Select Graph G(V, E)
2. Set V= n
3. Color all vertexes white
4. Find the centre of graph say $v_1 \varepsilon V$
5. Color v_1 Black
6. Color Grey to all $n(v_1)$
7. $V = V- v_1$
8. Set V= n -1
9. Select gray vertex $v_2 \varepsilon V$ having the largest associated white vertices
10. Color Black to v_2
11. Color Grey to all n(v2) except any black vertex in n(v2)
12. $V = V- v_2$
13. Repeat steps 9-12 till all vertices are black & Grey, no white vertices
14. If all black vertex are connected than this is CDS and hence MCDS
15. Test CDS to be MCDS
16. If removing any black vertices will not affecting the CDS property then CDS is MCDS.

17. Repeat step 15 till we reach to the conclusion of step 16
18. End

Conclusion:

Traditionally, most existing algorithms have two phases. The first phase is to construct maximal independent set (MIS) as dominating set (DS). The second is to construct a Steiner tree for connecting nodes in DS through adding to connectors. In this paper, a new algorithm AJK algorithm the greedy strategy is proposed to get a relatively optimal size of the CDS. It reduces the size of the CDS algorithm effectively, sacrificing some performance of time. And provide us MCDS.

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